Internet Appendix to "Horizon Effects in Average Returns: The Role of Slow Information Diffusion"*

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Section 1 of this internet appendix reviews the main results in the prior literature on iid measurement error and nonsynchronous trade. We also provide a new theoretical result, downward bias in short-horizon mean returns, in the nonsynchronous trade and partial price adjustment model. We show that the iid measurement error produces horizon effects qualitatively different than those observed empirically in value-weighted portfolios. The nonsynchronous trade and partial price adjustment models are able to qualitatively match features of the data, but realistic calibrations cannot quantitatively match the horizon effects observed in the data. We show that horizon effects in returns extend to coefficients from Fama and MacBeth (1973) regressions in Section 2, and provide computational details and additional empirical results in Section 3.

1. Horizon Effects in Standard Microstructure Models

We first review how iid measurement errors impact portfolio returns following Blume and Stambaugh (1983) and other literature. We then provide a theoretical analysis of how nonsynchronous trade and partial price adjustment impact portfolio mean returns. Both of these standard frictions imply that measurement error in observed portfolio returns is correlated with fundamentals, and in contrast to existing literature, both imply downward bias in the average short-horizon returns of value-weighted portfolios.

1.1. Prior Literature and iid Measurement Errors

Following Blume and Stambaugh (1983), suppose the observed price P_{it} of an asset at time t is centered around the fundamental value P_{it}^*, but subject to mean-zero noise:

$$P_{it} = P_{it}^*(1+\delta_{it}). \tag{1}$$

The observed return is an upward biased estimate of the true return

$$\mathbb{E}\left[R_{it}\right] = \mathbb{E}\left[\frac{P_{it}}{P_{it-1}}\right] = \mathbb{E}\left[\frac{P_{it}^*(1+\delta_{it})}{P_{it-1}^*(1+\delta_{it-1})}\right] = \mathbb{E}\left[\mu\frac{(1+\delta_{it})}{(1+\delta_{it-1})}\right] \approx \mu + \sigma^2\left(\delta_{it}\right), \tag{2}$$

where $\mu = \mathbb{E}\left[R_{it}^*\right]$ is the mean fundamental return. The size of the bias due to measurement error depends, to a first order approximation, on the variance of the mispricing.

Blume and Stambaugh (1983) show the bias in returns of value-weighted versus equal-weighted, rebalanced portfolios. The return means are:

$$\mathbb{E}\left[R_{VW,t}\right] \approx \mu + \sigma^2\left(\bar{\delta}_{it}\right), \tag{3}$$

$$\mathbb{E}\left[R_{EW,t}\right] \approx \mu + \bar{\sigma}^2\left(\delta_{it}\right), \tag{4}$$

where $\bar{\delta}$ is the average measurement error of the stocks in the portfolio and $\bar{\sigma}^2\left(\delta_{it}\right)$ is the average variance of mispricing across stocks. Diversification reduces the bias caused by bid-ask bounce in the value-weighted portfolio. By contrast, the equal-weighted, rebalanced portfolio has bias of the same magnitude as an individual stock.

Roll (1983) shows empirically that over longer holding periods, daily-rebalanced returns substantially exceed buy-and-hold returns. Conrad and Kaul (1993), Canina, Michaely, Thaler, and Womack (1998), Liu and Strong (2008), and Asparouhova, Bessembinder, and Kalcheva ("ABK", 2010, 2013) provide additional empirical analysis, and propose empirical workarounds to the bias caused by rebalancing portfolio weights.

1.2. Bias in Mean Returns Under Nonsynchronous Trade

We develop new results showing the bias in average returns under nonsynchronous trade. In contrast to the insights from existing literature, which focuses on the case where measurement error is uncorrelated with fundamentals, we find that average short-horizon returns are downward biased, and the bias is more severe under value weighting than equal weighting.

Reflecting the central idea of the literature on nonsynchronous trade,[1] consider a continuum of stocks, which for simplicity pay no dividends. A proportion π of the stocks, $0 \leq \pi \leq 1$, called *leaders* trade continuously and always fully incorporate all available information. The remaining stocks, called *laggers* are assumed to always trade for the last time of the day at instant $t + \alpha$, where $0 < \alpha < 1$; $\alpha = 0$ corresponds to market opening, and $\alpha = 1$ to market closing.

We decompose the fundamental return from t to $t+1$, denoted R^*_{t+1}, into the return

[1] Nonsynchronous trade is discussed by Fisher (1966), Scholes and Williams (1977), Cohen, Maier, Schwartz, and Whitcomb (1979), Atchison, Butler, and Simonds (1987), and Lo and MacKinlay (1990), among others.

from opening at t until time $t+\alpha$ and a return from $t+\alpha$ to closing at $t+1$:

$$R^*_{t+1} = R^*_{t:t+\alpha} \times R^*_{t+\alpha:t+1}. \tag{5}$$

We assume the fundamental returns are independent, and denote $\mu \equiv \mathbb{E}\left[R^*_t\right]$ for all t. Leader prices always reflect current fundamental value, and their returns satisfy $R_{leader,t} = R^*_t$ for all t. Lagger returns are affected by stale prices:

$$R_{lagger,t+1} = R^*_{t-1+\alpha:t} \times R^*_{t:t+\alpha}. \tag{6}$$

Both types of stocks have returns that are unbiased and uncorrelated across time:[2]

$$\begin{aligned} \mathbb{E}\left[R_{it}\right] &= \mu & (7) \\ \mathrm{Cov}\left[R_{it}, R_{i,t+1}\right] &= 0 \qquad i \in \{leader, lagger\}. & (8) \end{aligned}$$

The effects of nonsynchronous trade on average portfolio returns are given by:

Proposition 1 *The observed return on an equal-weighted portfolio is unbiased:*

$$\mathbb{E}\left[R_{EW,t}\right] = \mu.$$

The average return of a value-weighted portfolio is

$$\mathbb{E}\left[R_{VW,t}\right] = \mathbb{E}\left[\frac{1}{\pi + (1-\pi)/R^*_{t-1+\alpha:t}}\right]\left[\pi\mu + (1-\pi)\mu^{\alpha}\right] \leq \mu.$$

If the portfolio contains leader and lagger stocks, $\pi \in (0,1)$*, the observed value-weighted return is downward biased:*

$$\mathbb{E}\left[R_{VW,t}\right] < \mu.$$

The downward bias in the observed return of the value-weighted portfolio is caused by the use of stale weights. Following a high realized return at the end of the previous day, $R^*_{t-1+\alpha:t} > 0$, the lagger stocks will be underweighted relative to their fundamental values. At the same time, their observed return on the next day is expected to be high. The average observed single-period portfolio return will thus be understated relative to the fundamental return.

[2]In the models of Scholes and Williams (1977) and Lo and MacKinlay (1990), the observed returns of an individual stock have small negative autocorrelation due to their allowance that the time between price observations is random. For simplicity, we model the time between trades as fixed, and thus individual stocks have uncorrelated returns. Blume and Stambaugh (1983) and Lo and MacKinlay (1990) show that the magnitude of the autocorrelation in individual stock returns induced by permitting a random time between trades is small.

1.2.1. Implementation of Boudoukh, Richardson, and Whitelaw (1994)

To demonstrate the potential quantitative impact of nonsynchronous trading, we implement the model described in Section 3.2 of Boudoukh, Richardson, and Whitelaw (1994). For all stocks, observed prices equal their true values whenever they are traded, but are equal to their last trade value otherwise. Independent binomial random variables generate stock-specific trade. Within a portfolio, we assume four classes $j \in \{1, 2, 3, 4\}$ of stocks with weights $w_j \in \{0.05, 0.45, 0.45, 0.05\}$. Individual stocks in each class have the same no-trade probability $p_j \in \{0.1, 0.2, 0.8, 0.9\}$. Fundamental log returns are generated by

$$r_{it} = \mu_i + \Lambda_t + \varepsilon_{it}, \tag{9}$$

where Λ is a common independent normal factor with volatility $\sigma_\lambda = 0.15$, $\mu_i = 0.04$, and ε_{it} are independent normal random variables with $\sigma_i = \sigma_\lambda$, all in annual units. As in Boudoukh, Richardson, and Whitelaw (1994), time increments are hours, with six trading hours per day. All stock prices are initialized to be $1, and 4000 years of data are generated. Each class j in the equal-weighted portfolio is rebalanced daily, and in the value weighted case once every 2 years (512 days). This resets the market values of individual stocks periodically and prevents an individual stock from dominating the economy. Average simple returns from the subsequence of daily prices are calculated using standard techniques.

1.3. Bias in Mean Returns Under Partial Price Adjustment

Partial price adjustment models imply that some subset of securities only partially adjusts to fundamental value. The effects are qualitatively similar to nonsynchronous trade, but can be driven by more general types of market frictions.[3]

Consider a continuum of stocks, which again pay no dividends. On a given day, a proportion π of the stocks are leaders and incorporate all of the information from that day into the reported closing stock price. The remaining stocks are laggers and have a

[3] Important explanations include imperfections in information transmission, the existence of noise traders, and the sequential execution of large orders, as discussed for example by Holden and Subrahmanyam (1992), Brennan, Jegadeesh, and Swaminathan (1993), Foster and Viswanathan (1993), Badrinath, Kale, and Noe (1995), Klibanoff, Lamont, and Wizman (1998), Chordia and Swaminathan (2000), and Llorente, Michaely, Saar, and Wang (2002).

recorded closing price and return:

$$P_{lagger,t} = \alpha P_t^* + (1-\alpha) P_{t-1}^*, \tag{10}$$

$$R_{lagger,t} = \frac{\alpha + (1-\alpha)/R_t^*}{\alpha + (1-\alpha)/R_{t-1}^*} R_t^*. \tag{11}$$

We then show:

Proposition 2 *The average observed return on an individual stock has expectation*

$$\mathbb{E}\left[R_{lagger,t}\right] = \mu + (1-\alpha)\, Cov\left[\frac{1}{\alpha R_t^* + (1-\alpha)}, R_t^*\right] \leq \mu.$$

Under partial adjustment, $0 < \alpha < 1$, the observed mean is biased downward relative to the fundamental return: $\mathbb{E}\left[R_{lagger,t}\right] < \mu$.

For a portfolio composed of both leaders and laggers,

Proposition 3 *Observed equal- and value-weighted mean returns are*

$$\mathbb{E}\left[R_{EW,t}\right] = \mu + (1-\gamma)\, Cov\left[\frac{1}{\alpha R_t^* + (1-\alpha)}, R_t^*\right] \leq \mu$$

$$\mathbb{E}\left[R_{VW,t}\right] = \mu + (1-\gamma)\, Cov\left[\frac{1}{\gamma R_t^* + (1-\gamma)}, R_t^*\right] \leq \mu,$$

where $\gamma = 1 - (1-\pi)(1-\alpha)$ satisfies $0 \leq \gamma \leq 1$. If $0 < \gamma < 1$, both equal- and value-weighted returns are downward biased, and the bias in the value-weighted return is larger: $\mathbb{E}\left[R_{VW,t}\right] < \mathbb{E}\left[R_{EW,t}\right] < \mu$.

Again, value-weighted returns are exposed to a larger bias than equal-weighted returns. We arrive at these implications by considering specific error structures suggested by the literature on microstructure frictions, which requires departing from the assumption of iid measurement errors.

1.4. Horizon Effects in the Models

We now provide general formulas for the observed average n-period return under each of the models considered in this section. The formulas show that short-horizon return averages are more biased than appropriately rescaled long-horizon return averages.

Under measurement error, an approximation for the mean of a multi-period return is

$$\begin{aligned}\mathbb{E}\left(R_{i,t+1}\cdots R_{i,t+n}\right) &= \mathbb{E}\left(\frac{P^*_{it+n}(1+\delta_{it+n})}{P^*_{it}(1+\delta_{it})}\right) \\ &\approx \mu^n e^{\sigma^2(\delta_{it})} \\ &\equiv \mu^n \Delta_B. \end{aligned} \tag{12}$$

In the case of lead-lag effects, an approximation is

$$\begin{aligned}\mathbb{E}\left(R_{i,t+1}\cdots R_{i,t+n}\right) &\approx \mu^n\left[1-\frac{\xi(1-\xi)}{\mu(\xi\mu+1-\xi)^2}\sigma^2(R^*_{it})\right] \\ &\equiv \mu^n \Delta_A, \end{aligned} \tag{13}$$

where $\xi=\pi$ in the case of nonsynchronous trading and $\xi=\gamma$ in the case of partial price adjustment with value-weighting.[4] Both of equations (12) and (13) show that all of the microstructure effects we consider in Section 1 cause gross return averages to be biased by the proportion Δ_j, $j\in\{A,B\}$. Furthermore, the proportional bias in average gross returns is independent of the investing horizon.

The relative bias in average net returns is given by

$$\frac{\mu^n\Delta_j-\mu^n}{\mu^n-1}=\frac{\Delta_j-1}{1-\mu^{-n}}. \tag{14}$$

This expression is decreasing in n and shows that the proportional bias in the net simple return averages approaches Δ_j-1 at long horizons. When $\mu\approx 1$, as is typical for daily returns, the proportional bias in gross return means is significantly magnified.

Microstructure effects can be mitigated by rescaling gross return averages. Consider the estimate of m-period return mean given by

$$\bar{R}_{inm}\equiv\mathbb{E}\left(R_{i,t+1}\cdots R_{i,t+n}\right)^{m/n}=\left(\mu^n\Delta_j\right)^{m/n}=\mu^m\Delta_j^{m/n}. \tag{15}$$

The relative bias in this estimate is inversely proportional to n, showing that long-horizon return averages can be used to create unbiased estimates of short-horizon return means.

[4]The approximation follows from applying a first-order Taylor series expansion to the term $1/(\xi R_t+1-\xi)$ around $R_t=\mu$ in equation (28). A similar but notationally more cumbersome expression, not reported, applies to the case of partial price adjustment with equal-weighting.

An alternative technique to eliminate microstructure bias rescales means from different horizons. An estimate of the m-period mean can be obtained from the ratio

$$\frac{\mathbb{E}\left(R_{i,t+1}\cdots R_{i,t+n+m}\right)}{\mathbb{E}\left(R_{i,t+1}\cdots R_{i,t+n}\right)} = \frac{\mu^{n+m}\Delta_j}{\mu^n\Delta_j} = \mu^m. \tag{16}$$

In the models we consider, microstructure biases can be eliminated whenever $n \geq 1$. In practice more than one lag may be necessary to capture microstructure effects, and larger values of n may be required empirically for Equation (16) to hold.

Figure IA1 shows plots of average holding period return versus the return-period length as a diagnostic for each of the microstructure models in this Appendix, using simulated data from each model and the estimation method for $\bar{R}_{inm}$ described in section 3.1. In all cases, the equal-weighted, daily-rebalanced portfolios shown on the left-hand side of the figure suffer substantial biases. For buy-and-hold portfolios, shown on the right-hand side of the figure, iid measurement error (Panel A) does not cause a problem in rescaled returns. However, nonsynchronous trade and partial price adjustment (Panels B and C), give pronounced downward biases in short-horizon returns. All of these results are consistent with our analysis in this Appendix. The effects shown in Panels B and C can be magnified. Boudoukh, Richardson, and Whitelaw (1994) show that permitting heterogeneity in the degree to which different stocks are impacted by nonsynchronous trade increases portfolio autocorrelations. Following our analysis, this should also impact the magnitude of the bias in short-horizon average returns. Panel D shows the additional bias produced by the calibrated model of Boudoukh, Richardson, and Whitelaw (1994).

Figure IA1 conveniently summarizes all of the different biases in average returns that have been discussed in the prior literature, including those identified for the first time in this paper. The figure also shows that a simple procedure, plotting the rescaled average return over different holding period lengths, provides an effective tool for detecting the effects of microstructure frictions.

2. Fama-MacBeth Regressions

2.1. Horizon Effects in Fama-MacBeth Regressions

The choice of return horizon also impacts Fama and MacBeth (1973, FM) coefficients and their interpretation. Consider cross-sectional regressions:

$$R_{in,t} - 1 = a_{nt} + b_{nt} X_{it} + \epsilon_{int} \tag{17}$$

where for simplicity X_{it} is a univariate characteristic for stock i at time t, $R_{in,t}$ is an n-period gross return on stock i starting at time t, and ϵ_{int} is uncorrelated and mean zero. Fama (1976, p. 328) shows that the time-series of FM coefficients b_{nt} can be interpreted as payoffs on a zero-cost investment strategy with portfolio weights proportional to the characteristic.[5]

To see how the estimated FM coefficients change with time scale, consider estimating (17) using $n = 1$ and $n = 2$ period returns. The conditional expectation of a 1-period return is

$$\mathbb{E}\left(R_{in,t} \mid X_{it}\right) = 1 + a_1 + b_1 X_{it}. \tag{18}$$

The conditional expectation of the two-period return is

$$\begin{aligned} \mathbb{E}\left(R_{in,t} R_{in,t+1} \mid X_{it}\right) &= \mathbb{E}\left(1 + a_{1t} + a_{1t+1} + b_{1t} X_{it} + b_{1t+1} X_{it+1} \mid X_{it}\right) \\ &\quad + \mathbb{E}\left(a_{1t} a_{1t+1} \mid X_{it}\right) \\ &\quad + \mathbb{E}\left(a_{1t} b_{1t+1} X_{it+1} + a_{1t+1} b_{1t} X_{it} \mid X_{it}\right) \\ &\quad + \mathbb{E}\left(b_{1t} b_{1t+1} X_{it} X_{it+1}) \mid X_{it}\right). \end{aligned} \tag{19}$$

If the time-series of FM coefficients are serially independent and uncorrelated with the characteristic X_{it}, only the first term of Equation (19) is substantially different from zero. In this case, provided the characteristic X_{it} is approximately constant across periods, the coefficient means scale approximately linearly, i.e., $a_2 \approx 2a_1$ and $b_2 \approx 2b_1$, where a_n and b_n respectively denote the time-series averages of the regression coefficients a_{nt} and b_{nt}. Serial correlations in the time-series of FM coefficients can be driven by pricing frictions. For example, in the case where positive autocorrelations and cross-serial correlations are driven by heterogeneous information diffusion, we expect the

[5] For example, suppose X_{it} is a small stock indicator. Then, a_{nt} measures the realized return of a large stock portfolio, and b_{nt} represents the realized size premium.

short-horizon coefficients to be downward biased, $2a_1 < a_2$, $2b_1 < b_2$.[6] Generally, different types of frictions could lead to different patterns in the autocorrelations of the FM coefficients.

In recent work, ABK (2010) focus on the specific case of noise in prices that is uncorrelated with fundamentals. They show that weighting observations i by their lagged returns can correct the bias caused by iid noise in prices. The ABK weighting scheme would not work, however, to correct bias in FM coefficients caused by heterogenous information diffusion, as such a weighting scheme does not address the autocorrelation terms in (19). To account for both iid measurement error, as in ABK, and nonsynchronous price adjustment, one would need to use the ABK weights as well as choose a return horizon long enough to minimize the importance of autocorrelations and cross-autocorrelations in the FM coefficients.

3. Computational Details and Additional Results

3.1. Calculation of Rescaled Average Returns

One simple way to obtain an estimate of $\bar{R}_{inm}$ would be to partition the available data into $n-$period windows, calculate average buy-and-hold return over the windows, and raise the average to the power m/n. This approach suffers two important drawbacks. First, the result will depend on the particular placement of the partitions. Second, observations at the beginning or end of the sample are not used if they do not form a complete $n-$period window. We develop a procedure that overcomes both of those problems. It is based on rolling $n-$period windows and incorporates an iterative adjustment for the omitted observations at the beginning or end of the sample.

For the first iteration $i = 1$, we create an average using only periods t for which a complete n-period return is available. Specifically, let $R_{nt} = R_{t-n+1}...R_t$ and define

$$\bar{R}_n^{(i=1)} = \sum_{t=n}^{T} \frac{R_{nt}}{T-n+1},$$

where T is the sample size.

[6] For example, X_{it} inversely related to size ensures cross-serial correlations will be positive since large stocks typically lead small stocks.

For each iteration $i > 1$, we recalculate the average $\bar{R}_n^{(i)}$ using the entire dataset, and using the average $\bar{R}_n^{(i-1)}$ calculated in the previous round to rescale when an incomplete return is available. That is, we first calculate the rescaled returns

$$R_{nt}^i = (R_1...R_t)\frac{\bar{R}_n^{(i-1)}}{\bar{R}_t}, \qquad t < n$$

where the average t-period returns $\bar{R}_t$ have previously been calculated for $t < n$. We then recalculate the average n-period return

$$\bar{R}_n^{(i=1)} = \frac{\sum_{t=1}^T w_t R_{nt}^{(i)}}{\sum_{t=1}^T w_t},$$

where the weights $w_t = \max(n, t)/n$ are proportional to the number of periods available to calculate each return $R_{nt}^{(i)}$. We repeat this procedure until $\bar{R}_n^{(i=1)}$ converges.

The resulting average n-day buy-and-hold returns $\bar{R}_n$ can then be rescaled geometrically to an $m-$period horizon, $\bar{R}_{inm} = \left(\bar{R}_n\right)^{m/n}$. SAS code necessary to implement this procedure is available from the authors.

3.2. Linear versus Geometric Rescaling

Tables IA1 and IA2 replicate Tables 1 and 2 from the main text using linear rather than geometric rescaling. The results are very similar, confirming that compounding effects are small at the horizons we consider.

3.3. Horizon Effects in Other Style-Sorted Portfolios

Table IA3 compares average daily and monthly returns rescaled to a quarterly horizon for additional style portfolios. We consider portfolios formed on the basis of weekly reversal (Lehmann (1990)), asset growth (Cooper, Gulen, and Schill (2008)), accruals (Sloan (1996)), and stock issuance (Daniel and Titman (2006)). In all long-only portfolios, rescaled short-horizon returns understate the quarterly buy-and-hold returns.

3.4. Horizon Effects in the Fama-French and Momentum Factors

Table IA4 shows the effects of rescaling for the Fama-French and momentum factors.[7] The biases are generally small. For example, in the value-weighted market portfolio the

[7] We thank Ken French for making data for the size, value, and momentum factors available on his website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

rescaled daily returns understate quarterly buy-and-hold means by 0.07 per quarter. Horizon effects in the value factor are also negligible (0.06) as biases in the value and growth stocks offset one another. In the momentum factor, a substantial bias in the loser stock portfolio is only partially offset by the winners, giving a net bias of 0.22 quarterly. The size factor exhibits the largest horizon effects (0.31), stemming almost entirely from small stocks. The use of broad portfolios rather than the extreme deciles, value weighting, and the construction using NYSE breakpoints all contribute to the microstructure-induced biases in the Fama-French factors being modest.

3.5. Horizon Effects in Country and Regional MSCI Indices

From Datastream, we identify all available country and regional US-dollar-denominated MSCI indices. To be included in our sample, we require indices to have at least 10 years of valid data to produce our international return sample of 56 country and 49 regional indices.[8]

Tables IA5 and IA6 show that horizon effects are significant in country and regional indices. Across the 56 countries, the average difference between mean monthly returns geometrically rescaled to annual frequency (RS) and buy-and-hold annual return means (BH) is 2.79% per year. Consistent with our theoretical predictions, horizon effects are stronger in smaller, less liquid markets where information is likely to diffuse slowly. The average difference between BH and RS returns in countries classified by MSCI as Developed is 1.26% annually, whereas for the Emerging and Frontier markets the difference is 3.93% on average and exceeds 10% for several cases.

3.6. Changing Market Composition and Horizon Effects Over Time

A large literature, cited in footnote 6 of the paper, suggests that slow information diffusion remains important even in recent data. All else constant, the speed of information assimilation has increased over time. Empirically, it is difficult to disentangle the offsetting effects of higher volume or complexity of information and the speed of information diffusion. In this subsection we address this issue in the time series of

[8] MSCI maintains a number of regional indices (e.g., Europe Excluding Ireland) that closely overlap with other broader indices that we study (e.g., Europe). For brevity, we exclude such regional indices.

the U.S. market, by showing that the characteristics of the small stock portfolio have changed dramatically over time.

We find that after adjusting for the average growth of the U.S. market, small stocks today are significantly smaller than they were in the early sample. To show this, we first compute the cross-sectional percentile breakpoints of market capitalizations in 1926. We then impute what those percentiles breakpoints would be for every year of our sample had they grown as the overall market. In other words, each year, breakpoints change with the ex-dividend market return. If the distribution of market capitalizations only grew with market returns over time, we would expect the proportion of stocks in each group to be constant.

Figure IA2 shows how the proportion of stocks in each group actually evolves over time. By construction, the lines start at their respective percentiles (1, 10, 25, 50, 75, 90, and 99), and change as the distribution of market capitalizations in the stock universe changes. The plot on the left (right) assumes that the breakpoints grow with the value-weighted (equal-weighted) ex-dividend market return. Most importantly, in 2009, 38% of the stocks fall into the group that would have comprised the small stock decile in 1926 (left panel). In other words, there are a lot more very small companies today than at the beginning of our sample. This can also be seen in the two discrete shifts in the distribution as Amex (in 1962) and NASDAQ (in 1972) stocks are added to the CRSP universe. The 38% is a conservative estimate, since small stocks on average outperform the market. The right panel shows a much more dramatic effect when breakpoints are assumed to grow at the equal-weighted market return.

Figure IA3 shows that, when holding the firm size distribution fixed, horizon effects indeed become smaller over time. The test portfolio we use contains all stocks that fall between the 1st and 10th percentile breakpoints, which are obtained in 1926 and adjusted for market returns.[9] As in the paper (Figure 4), horizon effects are very strong between 1926 and 1949, and are much smaller from 1950 to 1969. Controlling for the firm size distribution, however, has a large impact on the estimated horizon effects after 1970. In the paper, we show that effects remain roughly constant at 6% annually in the second half of the sample. Fixing relative firm sizes, Figure IA3 shows effects of

[9] We did not base our analysis on the smallest company in 1926 (or percentile 0) to avoid inferences being affected by one outlier.

around 4% in 1970-1989, and 1% from 1990-2009.

3.7. Empirical Evidence on Horizon Effects in FM Regressions

Let n denote the return measurement interval, which may be monthly ($n = 1$) or quarterly ($n = 3$). To demonstrate the impact of the measurement interval n on FM regression coefficients we consider size-based regressions using either dummy variables or a continuous-valued characteristic. In all cross-sectional regressions, we use weighted least squares with two different sets of weights that follow from the recommendations of ABK (2010): 1) standard value weighting, and 2) weighting by the most recent one-month gross return, which can be viewed as a modification of equal weighting. ABK show that both weighting methods are robust to measurement errors in individual stocks that are uncorrelated with movements in fundamentals.

Consistent with our prior theoretical treatment of nonsynchronous trade and partial price adjustment, we expect that in many types of cross-sectional regressions, including the size-based regressions that we focus on in the remainder of this section, measurement errors in individual stock returns will be correlated with movements in fundamentals. The weighting schemes recommended by ABK do not address these types of measurement errors. We therefore expect to find biases in FM regression coefficients estimated from short-horizon returns. Following from equation (21), the direction and magnitude of the biases should be linked to patterns in the autocorrelations and cross-serial correlations of FM regression coefficients, driven in turn by differences in the speed of adjustment to fundamentals. At longer return measurement intervals where nonsynchronicity is less of an issue, we expect serial correlations in the time-series of FM coefficients to diminish and the FM coefficient means should be unbiased.

We first consider the dummy variable regression:

$$R_{int} - 1 = a_{Snt} d_{1,it} + a_{Mnt} d_{2:9,it} + a_{Lnt} d_{10,it} + \epsilon_{int}, \tag{20}$$

where $d_{1,it}$, $d_{2:9,it}$, and $d_{10,it}$ are dummy variables indicating membership of stock i in size decile groupings according to market capitalization at the beginning of the year. The specific decile groupings we use are: small ("S", decile = 1), medium ("M", $2 \leq$ decile ≤ 9), or large ("L", decile = 10). With these groupings, the estimated coefficients a_{Snt}, a_{Mnt}, and a_{Lnt} have interpretations as portfolio returns for small, medium, and large deciles of stocks. The estimated coefficients can therefore be directly related to our previous results on the scaling of average returns in style portfolios.

Equivalent to (20), we omit the dummy variable for the large category of stocks, and instead use a constant in the regression:

$$R_{int} - 1 = a_{Lnt} + a_{S-L,nt} d_{1,it} + a_{M-L,nt} d_{2:9,it} + \epsilon_{int}. \tag{21}$$

The constant now picks up the returns on large stocks, and the other estimated coefficients $a_{S-L,nt}$ and $a_{M-L,nt}$ reflect return differences relative to large stocks. In comparison with (20), the regression (21) appears more typical of a standard FM regression due to the presence of a constant and the interpretation of the coefficients as payoffs on long-short portfolios.

Finally, we consider a size-based characteristic regression with a continuously-valued dependent variable:

$$R_{int} - 1 = a_{0nt} + a_{ME,nt} ME_{it} + \eta_{int}, \tag{22}$$

where ME_{it} is the demeaned logarithm of market capitalization at the beginning of the year. This regression is most typical of standard FM regressions, and also closely relates to the dummy regression (21). Examining the regressions (20-22) at the the horizons $n = 1$ and $n = 3$ will illustrate how scaling in average returns carries over to scaling in FM coefficients.

Table IA7 reports coefficients from these models. Panel A shows the results from the dummy variable regression (20). Consistent with our prior results, the monthly regression coefficients rescaled to a quarterly horizon (RS) are downward biased relative to the quarterly regression coefficients (BH). The rescaling bias is most severe for the coefficient on the small stock dummy and least severe for the coefficient on the large stock dummy. The magnitude of the bias is driven by the autocorrelation patterns in the one-month regression coefficients. The small stock regression coefficient is very persistent (0.22 autocorrelation in both return-weighted and size-weighted regressions), while the autocorrelation in the large stock coefficient is milder (0.13 and 0.10 in return- and size-weighted regressions). We also observe that the large stock coefficients substantially lead the small stock coefficients (cross-serial correlations of 0.28 and 0.25) while the effect of small stocks leading large stocks is less important (cross-serial correlations of 0.07 and 0.07). Serial correlations in the estimated coefficients are substantially diminished in the quarterly ($n = 3$) regressions.

Panel B shows results from the dummy variable regression (21), where a constant is included in the regression and the large stock dummy is omitted. Recalling our theoretical analysis and the interpretation of Fama-MacBeth coefficients as portfolio returns, we again expect the RS coefficients to be downward biased relative to the BH coefficients. The bias should be more severe for characteristics associated with stronger microstructure frictions, which will tend to cause positive autocorrelations in the time-series of Fama-MacBeth coefficients. The empirical results are consistent with these predictions. Using value weighting, the coefficient α_{S-L} equals 3.04 percent quarterly when calculated from quarterly returns (BH), but only 2.28 percent per quarter when rescaled from the monthly estimate (RS). Following from equation (21), the downward bias of 76 basis points per quarter is driven by the more severe persistence of small stocks relative to large stocks, and the strong cross-serial correlation effect of large stocks leading small stocks.[10] In contrast, the FM regression coefficients calculated from quarterly returns have autocorrelations and cross-correlations much closer to zero.

Results are qualitatively similar for the regressions based on the size characteristic (20), as shown in Panel C.[11] The time-series of FM coefficients estimated from monthly returns are significantly positively correlated, but close to zero when calculated from quarterly returns, and the rescaled monthly FM coefficients are downward biased relative to the quarterly coefficients. As implied by our previous analysis, whenever slow price adjustment causes substantial positive autocorrelations in portfolio returns, the time-series of FM coefficients are positively correlated and their time-series mean is downward biased.

These results show that in the context of Fama-MacBeth regressions, researchers should be cognizant of the impact of microstructure frictions. Researchers are gen-

[10]For comparison, averages reported in Table IA1 indicate a BH size premium of $5.12 - 2.49 = 2.63$ percent, a rescaled estimate of $4.37 - 2.43 = 1.94$ percent, and a downward bias from rescaling of $2.63 - 1.94 = 0.69$ percent. The remaining differences can be explained by small variations such as the use of calendar months in Table IA7 versus 21-day trading months in Table IA1.

[11]To determine the economic magnitude of the bias, we can substitute the difference in log market capitalization of small versus large stocks, which is on the order of 7, to determine a bias of around 20 basis points per quarter. Non-linearities in the size effect contribute to differences in the magnitudes of the rescaling biases shown in Panels B and C.

erally aware that autocorrelations in FM coefficients bias standard errors and hence t-statistics associated with the time-series mean of the FM coefficients. Researchers commonly use Newey-West (1987) adjustments to account for such time-series correlation. Our analysis shows that researchers should also be concerned about how the estimated FM coefficient means are biased in the presence of serial correlation caused by microstructure frictions.

4. Proofs

Proof of Proposition 1

The proof for the equal weighted result is straightforward since returns on both liquid and illiquid stocks are unbiased

$$\begin{aligned} \mathbb{E}\left[R_{EW,t}\right] &= \mathbb{E}\left[\pi R_{leader,t} + (1-\pi)R_{lagger,t}\right] \\ &= \pi\mu + (1-\pi)\mu = \mu. \end{aligned} \tag{23}$$

For the value weighted case, note that the true weights of liquid and illiquid stocks are π and $1-\pi$ respectively. The observed weight for liquid stocks remains π since liquid stocks trade continuously, but the observed weight on illiquid stocks is $(1-\pi)/R_{t-1+\alpha:t}$ because their last trade on record occurred at the price level $1/R_{t-1+\alpha:t}$.

$$\begin{aligned} \mathbb{E}\left[R_{VW,t}\right] &= \mathbb{E}\left[\left(\frac{1}{\pi + (1-\pi)/R^*_{t-1+\alpha:t}}\right)\left(\pi R_{leader,t} + \frac{1-\pi}{R^*_{t-1+\alpha:t}} R_{lagger,t}\right)\right] \\ &= \mathbb{E}\left[\left(\frac{1}{\pi + (1-\pi)/R^*_{t-1+\alpha:t}}\right)\left(\pi R_{leader,t} + \frac{1-\pi}{R^*_{t-1+\alpha:t}}\left(R^*_{t-1+\alpha:t}R^*_{t:t+\alpha}\right)\right)\right] \\ &= \mathbb{E}\left[\frac{1}{\pi + (1-\pi)/R^*_{t-1+\alpha:t}}\right]\left[\pi\mu + (1-\pi)\mathbb{E}\left(R^*_{t:t+\alpha}\right)\right]. \end{aligned} \tag{24}$$

An application of Jensen's inequality shows that

$$\mathbb{E}\left[\frac{1}{\pi + (1-\pi)/R^*_{t-1+\alpha:t}}\right] < \frac{1}{\pi + (1-\pi)/\mathbb{E}\left[R^*_{t-1+\alpha:t}\right]} \tag{25}$$

and therefore

$$\mathbb{E}\left[R_{VW,t}\right] < \frac{\mu}{\pi\mu + (1-\pi)\mathbb{E}\left(R^*_{t:t+\alpha}\right)}\left[\pi\mu + (1-\pi)\mathbb{E}\left(R^*_{t:t+\alpha}\right)\right] = \mu. \tag{26}$$

Proof of Proposition 2

The expected observed return on the laggers is given by

$$\begin{aligned}\mathbb{E}\left[R_{lagger,t}\right] &= \mathbb{E}\left[\frac{1}{\alpha+(1-\alpha)/R^*_{t-1}}\left(\alpha R^*_t+(1-\alpha)\right)\right] \\ &= \mathbb{E}\left[\frac{R^*_t}{\alpha R^*_t+(1-\alpha)}\right]\mathbb{E}\left[\alpha R^*_t+(1-\alpha)\right] \qquad (27)\end{aligned}$$

where the second equality is due to the assumption that R_t is *iid* over time.

To complete the proof, we write the expected stock return in terms of covariances:

$$\begin{aligned}\mathbb{E}\left[R_{lagger,t}\right] &= \mathbb{E}\left[\frac{R^*_t\left(\alpha R^*_t+(1-\alpha)\right)}{\alpha R^*_t+(1-\alpha)}\right]-\mathrm{Cov}\left(\frac{R^*_t}{\alpha R^*_t+(1-\alpha)},\alpha R^*_t+(1-\alpha)\right) \\ &= \mu-\mathrm{Cov}\left(\frac{\alpha R^*_t}{\alpha R^*_t+(1-\alpha)},R^*_t\right) \\ &= \mu-\mathrm{Cov}\left(1-\frac{(1-\alpha)}{\alpha R^*_t+(1-\alpha)},R^*_t\right) \\ &= \mu+(1-\alpha)\,\mathrm{Cov}\left(\frac{1}{\alpha R^*_t+(1-\alpha)},R^*_t\right). \qquad (28)\end{aligned}$$

The covariance terms is negative for $\alpha > 0$.

Proof of Proposition 3

We consider the portfolio of liquid and illiquid stocks. We assume for convenience that $P_{lagger,0} = P_{leader,0}$, and suppose that the portfolio is both equal and value-weighted at date zero. The claim for the equal weighted portfolio follows immediately from Proposition 2:

$$\begin{aligned}\mathbb{E}\left[R_{EW,t}\right] &= \mathbb{E}\left[\pi R_{leaders,t}+(1-\pi)\,R_{laggers,t}\right] \\ &= \pi\mu+(1-\pi)\left(\mu+(1-\alpha)\,\mathrm{Cov}\left(\frac{1}{\alpha R^*_t+(1-\alpha)},R^*_t\right)\right) \\ &= \mu+(1-\pi)(1-\alpha)\,\mathrm{Cov}\left(\frac{1}{\alpha R^*_t+(1-\alpha)},R^*_t\right) \\ &= \mu+(1-\gamma)\,\mathrm{Cov}\left(\frac{1}{\alpha R^*_t+(1-\alpha)},R^*_t\right), \qquad (29)\end{aligned}$$

where $\gamma = 1-(1-\pi)(1-\alpha)$ denotes the portfolio liquidity.

For the value-weighted portfolio of leaders and laggers, note that total market value of leaders is the full information price, $P_{leader,t} = P^*_t$, and the value of laggers is $P_{lagger,t} =$

$\alpha P_t^* + (1-\alpha)P_{t-1}^*$. The value-weighted portfolio share in leaders is

$$\begin{aligned} W_{leader,t}^{\pi} &= \frac{\pi P_t^*}{\pi P_t^* + (1-\pi)\left(\alpha P_t^* + (1-\alpha)\, P_{t-1}^*\right)} \\ &= \frac{\pi}{\pi + (1-\pi)\left(\alpha + (1-\alpha)\,/R_t^*\right)} \\ &= \frac{\pi}{\gamma + (1-\gamma)\,/R_t^*}. \end{aligned} \tag{30}$$

For the value weighted return, note that

$$\begin{aligned} (1 - W_{t-1}^{\pi})R_{lagger,t} &= \frac{(1-\pi)\left(\alpha + (1-\alpha)\,/R_{t-1}^*\right)}{\pi + (1-\pi)\left(\alpha + (1-\alpha)\,/R_{t-1}^*\right)} \times \frac{\alpha + (1-\alpha)\,/R_t^*}{\alpha + (1-\alpha)/R_{t-1}^*} R_t^* \\ &= \frac{(1-\pi)\left(\alpha + (1-\alpha)\,/R_t^*\right)}{\pi + (1-\pi)\left(\alpha + (1-\alpha)\,/R_{t-1}^*\right)} R_t^* \\ &= \frac{\gamma + (1-\gamma)\,/R_t^* - \pi}{\gamma + (1-\gamma)\,/R_{t-1}^*} R_t^*. \end{aligned} \tag{31}$$

The value weighted portfolio return is therefore

$$R_{VW,t} = W_{t-1}^{\pi} R_t^* + (1 - W_{t-1}^{\pi})R_{lagger,t} = \frac{\gamma + (1-\gamma)\,/R_t^*}{\gamma + (1-\gamma)\,/R_{t-1}^*} R_t^*. \tag{32}$$

The value-weighted portfolio return has the same form as the return of the laggers, but with liquidity parameter γ instead of α. An application of Proposition 2 completes the proof to Proposition 3.

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Table IA1. HORIZON EFFECTS IN STYLE PORTFOLIOS, LINEAR RESCALING

Performance	Holding Horizon					Holding Horizon				
Metric	1 day	1 mo	3 mo	6 mo	1 year	1 day	1 mo	3 mo	6 mo	1 year
A. CRSP Value-Weighted Index										
Daily	0.04	0.84	2.52	5.04	10.08					
Monthly		0.86	2.59	5.19	10.38					
Quarterly			2.62	5.24	10.48					
Semi-Annual				5.33	10.65					
Annual					10.97					
B. Market Capitalization, Initially Value-Weighted										
	Big					Small				
Daily	0.04	0.80	2.40	4.81	9.62	0.06	1.25	3.76	7.52	15.03
Monthly		0.81	2.43	4.85	9.70		1.46	4.37	8.75	17.50
Quarterly			2.49	4.97	9.95			5.12	10.23	20.46
Semi-Annual				5.06	10.12				10.43	20.86
Annual					10.49					22.65
C. Market Capitalization, Initially Equal-Weighted										
	Big					Small				
Daily	0.04	0.81	2.43	4.86	9.71	0.07	1.40	4.21	8.42	16.84
Monthly		0.83	2.50	4.99	9.99		1.63	4.90	9.80	19.60
Quarterly			2.58	5.15	10.30			5.73	11.45	22.90
Semi-Annual				5.19	10.37				11.78	23.55
Annual					10.69					25.87
D. Book-to-Market, Initially Value-Weighted										
	Value					Growth				
Daily	0.05	1.15	3.46	6.93	13.85	0.04	0.77	2.30	4.61	9.22
Monthly		1.20	3.59	7.17	14.35		0.78	2.35	4.69	9.39
Quarterly			3.64	7.28	14.57			2.42	4.84	9.68
Semi-Annual				7.31	14.63				4.96	9.92
Annual					15.37					10.52
E. Book-to-Market, Initially Equal-Weighted										
	Value					Growth				
Daily	0.06	1.35	4.04	8.09	16.18	0.02	0.48	1.43	2.86	5.72
Monthly		1.49	4.46	8.91	17.83		0.59	1.78	3.57	7.13
Quarterly			4.70	9.40	18.79			1.97	3.94	7.87
Semi-Annual				9.65	19.30				3.82	7.65
Annual					19.91					7.82
F. Momentum, Initially Value-Weighted										
	Winners					Losers				
Daily	0.07	1.50	4.50	9.00	18.00	-0.01	-0.15	-0.46	-0.91	-1.82
Monthly		1.51	4.54	9.09	18.18		0.03	0.09	0.18	0.37
Quarterly			4.72	9.45	18.89			0.37	0.75	1.50
Semi-Annual				9.76	19.52				0.73	1.46
Annual					20.26					1.82
G. Momentum, Initially Equal-Weighted										
	Winners					Losers				
Daily	0.08	1.70	5.10	10.20	20.40	0.03	0.67	2.00	4.01	8.01
Monthly		1.80	5.41	10.82	21.65		1.00	3.01	6.02	12.04
Quarterly			5.74	11.49	22.98			3.47	6.95	13.90
Semi-Annual				12.05	24.11				7.21	14.41
Annual					24.69					14.97

Notes: This table reports average buy-and-hold returns (BH, on the diagonal) as well as linearly rescaled short horizon returns (RS, off-diagonal). Returns are calculated using periods of $n = 1, 21, 63, 126,$ and 252 days corresponding to Daily, Monthly, Quarterly, Semi-Annual, and Annual frequencies, and are scaled to the corresponding Holding Horizon. For all calculations, Panel A uses the value-weighted CRSP index, and the remaining use either initially equally-weighed (IEW) or initially value-weighted (IVW) daily returns for size, value, and momentum portfolios. A complete description of the portfolios is provided in Appendix B in the main text.

Table IA2. Horizon Effects: Significance and Decomposition, Linear Rescaling

A. Daily Performance Metric, Quarterly Holding Horizon

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
	H	L	HL	H	L	HL	H	L	HL	H	L	HL
RS	2.43	4.21	-1.78	4.04	1.43	2.61	5.10	2.00	3.10	5.39	2.87	2.52
BH	2.58	5.73	-3.15	4.70	1.97	2.73	5.74	3.47	2.27	7.83	3.07	4.76
RS-BH	-0.15	-1.51	1.37	-0.65	-0.54	-0.12	-0.64	-1.47	0.83	-2.44	-0.20	-2.24
	[-2.67]	[-3.16]	[3.12]	[-6.87]	[-4.67]	[-1.17]	[-4.89]	[-4.09]	[2.62]	[-3.91]	[-5.26]	[-3.73]
σ_{RS}	1.09	1.45		0.88	1.22		1.35	1.47		1.47	0.96	
ρ_{RS}	0.08	0.13		0.25	0.19		0.15	0.25		0.23	0.13	
VR	1.33	2.76		3.30	2.14		1.88	2.90		3.63	1.57	
RS/BH	0.94	0.74		0.86	0.73		0.89	0.58		0.69	0.94	
ν_{in}^{net}	0.95	0.79		0.88	0.73		0.91	0.62		0.75	0.94	

	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
	H	L	HL	H	L	HL	H	L	HL	H	L	HL
RS	3.03	3.02	0.01	3.03	2.62	0.42	3.13	2.90	0.24	3.48	1.29	2.20
BH	3.80	4.31	-0.50	4.52	2.79	1.73	3.54	3.31	0.23	3.82	2.25	1.57
RS-BH	-0.78	-1.29	0.51	-1.49	-0.18	-1.31	-0.40	-0.41	0.01	-0.34	-0.96	0.63
	[-5.14]	[-4.09]	[2.66]	[-4.61]	[-5.43]	[-4.36]	[-5.00]	[-3.46]	[0.09]	[-5.50]	[-5.02]	[3.84]
σ_{RS}	1.30	1.41		1.73	0.66		0.92	1.48		0.87	1.16	
ρ_{RS}	0.18	0.21		0.20	0.17		0.20	0.08		0.17	0.27	
VR	2.30	2.77		2.40	2.06		2.29	1.52		2.17	3.20	
RS/BH	0.80	0.70		0.67	0.94		0.89	0.88		0.91	0.57	
ν_{in}^{net}	0.81	0.73		0.70	0.95		0.90	0.89		0.92	0.58	

B. Monthly Performance Metric, Quarterly Holding Horizon

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
	H	L	HL	H	L	HL	H	L	HL	H	L	HL
RS	2.50	4.90	-2.40	4.46	1.78	2.67	5.41	3.01	2.40	6.62	2.99	3.63
BH	2.58	5.73	-3.15	4.70	1.97	2.73	5.74	3.47	2.27	7.83	3.07	4.76
RS-BH	-0.08	-0.82	0.75	-0.24	-0.19	-0.06	-0.33	-0.47	0.13	-1.21	-0.08	-1.13
	[-1.80]	[-2.33]	[2.31]	[-3.10]	[-1.89]	[-0.66]	[-3.37]	[-1.92]	[0.62]	[-2.68]	[-2.70]	[-2.59]
σ_{RS}	5.39	9.18		6.49	7.42		7.53	10.41		10.84	5.12	
ρ_{RS}	0.08	0.22		0.29	0.14		0.15	0.20		0.23	0.10	
VR	1.14	1.44		1.27	1.21		1.26	1.21		1.41	1.15	
RS/BH	0.97	0.86		0.95	0.91		0.94	0.87		0.85	0.97	
ν_{in}^{net}	0.98	0.90		0.96	0.91		0.96	0.90		0.90	0.98	

	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
	H	L	HL	H	L	HL	H	L	HL	H	L	HL
RS	3.49	3.76	-0.28	4.06	2.73	1.33	3.37	3.11	0.26	3.70	1.98	1.72
BH	3.80	4.31	-0.50	4.52	2.79	1.73	3.54	3.31	0.23	3.82	2.25	1.57
RS-BH	-0.32	-0.54	0.22	-0.46	-0.06	-0.40	-0.17	-0.19	0.02	-0.13	-0.27	0.15
	[-2.97]	[-2.42]	[1.56]	[-1.69]	[-2.34]	[-1.56]	[-3.14]	[-2.49]	[0.48]	[-2.82]	[-1.52]	[0.93]
σ_{RS}	8.03	9.41		11.31	4.01		5.72	7.73		5.37	8.64	
ρ_{RS}	0.23	0.19		0.19	0.08		0.17	0.12		0.13	0.23	
VR	1.26	1.30		1.17	1.18		1.25	1.17		1.19	1.21	
RS/BH	0.92	0.87		0.90	0.98		0.95	0.94		0.97	0.88	
ν_{in}^{net}	0.93	0.90		0.92	0.98		0.96	0.95		0.98	0.89	

Notes: This table reports and decomposes the horizon effects in the initially equally weighted buy-and-hold style portfolios. In Panel A, the rescaled (RS) performance measures are obtained from daily returns and compared to quarterly buy-and-hold (BH) returns. Panel B compares monthly RS returns to quarterly BH returns. RS returns are computed by linearly rescaling the average daily (Panel A) or monthly (Panel B) portfolio return to a quarterly frequency. BH returns are computed as average of n-day returns, where $n = 1, 21, 63$ corresponds to daily, monthly, and annual periods, respectively. t-statistics for the difference between RS and BH returns are shown in square brackets. Also reported are the standard deviation σ_{RS} and autocorrelation ρ_{RS} of daily (Panel A) and monthly (Panel B) returns, the relevant variance ratios (VR), as well as the empirically measured bias (RS/BH) and its analytical approximation ν_{in}^{net} from Proposition 4 in the main text. A complete description of the portfolios is provided in Appendix B in the main text.

Table IA3. Horizon Effects: Significance and Decomposition, Additional Style Portfolios

	Weekly reversal			Asset growth			Accruals			Issuance		
	H	L	HL	H	L	HL	H	L	HL	H	L	HL
A. Daily Performance Metric, Quarterly Holding Horizon, Geometric Rescaling												
RS	6.60	1.10	5.51	1.44	3.80	-2.35	2.08	3.54	-1.47	1.57	4.16	-2.59
BH	7.54	1.77	5.76	1.87	4.64	-2.77	2.68	4.20	-1.52	2.04	4.34	-2.30
RS-BH	-0.93	-0.68	-0.26	-0.43	-0.85	0.42	-0.60	-0.66	0.06	-0.47	-0.18	-0.29
	[-3.22]	[-2.43]	[-1.47]	[-4.79]	[-5.83]	[3.92]	[-5.47]	[-5.47]	[1.00]	[-4.61]	[-4.28]	[-3.60]
σ_{RS}	1.35	1.36		1.15	1.11		1.13	1.18		1.15	0.79	
ρ_{RS}	0.19	0.12		0.18	0.25		0.21	0.18		0.19	0.11	
VR	2.32	2.02		2.05	3.10		2.50	2.50		2.16	1.91	
RS/BH	0.88	0.62		0.77	0.82		0.77	0.84		0.77	0.96	
ν_{in}^{net}	0.89	0.66		0.76	0.82		0.77	0.84		0.76	0.96	
B. Monthly Performance Metric, Quarterly Holding Horizon, Geometric Rescaling												
RS	7.15	1.49	5.66	1.74	4.41	-2.67	2.50	4.01	-1.51	1.93	4.26	-2.33
BH	7.54	1.77	5.76	1.87	4.64	-2.77	2.68	4.20	-1.52	2.04	4.34	-2.30
RS-BH	-0.39	-0.28	-0.11	-0.13	-0.24	0.10	-0.19	-0.20	0.01	-0.11	-0.08	-0.03
	[-1.80]	[-1.30]	[-0.72]	[-2.02]	[-1.76]	[1.06]	[-1.96]	[-2.01]	[0.25]	[-1.19]	[-2.42]	[-0.41]
σ_{RS}	8.42	7.97		6.93	8.08		7.41	7.72		7.24	4.46	
ρ_{RS}	0.10	0.16		0.13	0.24		0.16	0.19		0.15	0.18	
VR	1.25	1.23		1.19	1.23		1.23	1.22		1.15	1.25	
RS/BH	0.95	0.84		0.93	0.95		0.93	0.95		0.95	0.98	
ν_{in}^{net}	0.96	0.88		0.92	0.95		0.93	0.95		0.94	0.98	
C. Daily Performance Metric, Quarterly Holding Horizon, Linear Rescaling												
RS	6.40	1.09	5.31	1.43	3.73	-2.29	2.06	3.48	-1.43	1.56	4.08	-2.52
BH	7.54	1.77	5.76	1.87	4.64	-2.77	2.68	4.20	-1.52	2.04	4.34	-2.30
RS-BH	-1.14	-0.68	-0.46	-0.44	-0.91	0.48	-0.63	-0.72	0.10	-0.49	-0.26	-0.22
	[-3.53]	[-2.42]	[-2.31]	[-4.89]	[-5.86]	[4.06]	[-5.58]	[-5.74]	[1.63]	[-4.75]	[-5.18]	[-2.65]
σ_{RS}	1.35	1.36		1.15	1.11		1.13	1.18		1.15	0.79	
ρ_{RS}	0.19	0.12		0.18	0.25		0.21	0.18		0.19	0.11	
VR	2.32	2.02		2.05	3.10		2.50	2.50		2.16	1.91	
RS/BH	0.85	0.62		0.77	0.80		0.77	0.83		0.76	0.94	
ν_{in}^{net}	0.89	0.66		0.76	0.82		0.77	0.84		0.76	0.96	
D. Monthly Performance Metric, Quarterly Holding Horizon, Linear Rescaling												
RS	6.98	1.48	5.50	1.73	4.34	-2.62	2.47	3.95	-1.48	1.92	4.21	-2.29
BH	7.54	1.77	5.76	1.87	4.64	-2.77	2.68	4.20	-1.52	2.04	4.34	-2.30
RS-BH	-0.55	-0.29	-0.26	-0.14	-0.30	0.16	-0.21	-0.25	0.04	-0.12	-0.14	0.01
	[-2.30]	[-1.31]	[-1.61]	[-2.16]	[-2.18]	[1.56]	[-2.17]	[-2.49]	[0.95]	[-1.35]	[-3.65]	[0.19]
σ_{RS}	8.42	7.97		6.93	8.08		7.41	7.72		7.24	4.46	
ρ_{RS}	0.10	0.16		0.13	0.24		0.16	0.19		0.15	0.18	
VR	1.25	1.23		1.19	1.23		1.23	1.22		1.15	1.25	
RS/BH	0.93	0.84		0.92	0.94		0.92	0.94		0.94	0.97	
ν_{in}^{net}	0.96	0.88		0.92	0.95		0.93	0.95		0.94	0.98	

Notes: This table reports and decomposes the horizon effects in the initially equally weighted buy-and-hold style portfolios. In Panels A and C, the rescaled (RS) performance measures are obtained from daily returns and compared to quarterly buy-and-hold (BH) returns. Panels B and D compare monthly RS returns to quarterly BH returns. RS returns are computed by geometrically (Panels A and B) or linearly (Panels C and D) rescaling the average daily (Panels A and C) or monthly (Panels B and C) portfolio return to a quarterly frequency. BH returns are computed as average of n-day returns, where $n = 1, 21, 63$ corresponds to daily, monthly, and annual periods, respectively. t-statistics for the difference between RS and BH returns are shown in square brackets, and for the case of geometric rescaling are calculated using the Delta method described in the Appendix in the main text . Also reported are the standard deviation σ_{RS} and autocorrelation ρ_{RS} of daily (Panel A) and monthly (Panel B) returns, the relevant variance ratios (VR), as well as the empirically measured bias (RS/BH) and its analytical approximation ν_{in}^{net} from Proposition 4 in the main text. A complete description of the portfolios is provided in Appendix B in the main text.

Table IA4. Horizon Effects: Significance and Decomposition, Factor Portfolios

	CRSP	Size			Value			Momentum		
	Index	H	L	HL	H	L	HL	H	L	HL
A. Daily Performance Metric, Quarterly Holding Horizon, Geometric Rescaling										
RS	2.55	3.26	2.81	0.45	3.63	2.36	1.27	4.49	0.51	3.97
BH	2.62	3.56	2.81	0.75	3.76	2.55	1.21	4.66	0.90	3.76
RS-BH	-0.07	-0.30	0.01	-0.31	-0.13	-0.19	0.06	-0.17	-0.39	0.22
	[-1.37]	[-4.37]	[0.13]	[-6.39]	[-2.65]	[-3.12]	[1.33]	[-2.22]	[-2.30]	[1.40]
σ_{RS}	1.07	0.98	0.98		0.95	1.05		1.31	1.60	
ρ_{RS}	0.08	0.15	0.05		0.09	0.13		0.11	0.13	
VR	1.20	2.01	1.00		1.47	1.55		1.33	1.50	
RS/BH	0.97	0.92	1.00		0.96	0.93		0.96	0.57	
ν_{in}^{net}	0.97	0.91	1.00		0.96	0.92		0.96	0.55	
B. Monthly Performance Metric, Quarterly Holding Horizon, Geometric Rescaling										
RS	2.62	3.45	2.79	0.66	3.70	2.48	1.22	4.56	0.78	3.78
BH	2.62	3.56	2.81	0.75	3.76	2.55	1.21	4.66	0.90	3.76
RS-BH	0.00	-0.10	-0.01	-0.09	-0.06	-0.07	0.01	-0.10	-0.12	0.02
	[-0.06]	[-2.14]	[-0.49]	[-2.49]	[-1.75]	[-1.54]	[0.29]	[-1.64]	[-1.18]	[0.22]
σ_{RS}	5.33	5.75	4.38		4.89	5.61		6.42	8.45	
ρ_{RS}	0.09	0.15	0.00		0.13	0.08		0.03	0.09	
VR	1.01	1.21	1.05		1.17	1.15		1.16	1.13	
RS/BH	1.00	0.97	1.00		0.98	0.97		0.98	0.86	
ν_{in}^{net}	1.00	0.97	0.99		0.98	0.97		0.98	0.85	
C. Daily Performance Metric, Quarterly Holding Horizon, Linear Rescaling										
RS	2.52	3.21	2.77	0.43	3.56	2.33	1.23	4.39	0.51	3.88
BH	2.62	3.56	2.81	0.75	3.76	2.55	1.21	4.66	0.90	3.76
RS-BH	-0.10	-0.35	-0.03	-0.32	-0.19	-0.22	0.02	-0.27	-0.39	0.12
	[-1.92]	[-4.86]	[-0.76]	[-6.64]	[-3.49]	[-3.37]	[0.48]	[-3.00]	[-2.30]	[0.78]
σ_{RS}	1.07	0.98	0.98		0.95	1.05		1.31	1.60	
ρ_{RS}	0.08	0.15	0.05		0.09	0.13		0.11	0.13	
VR	1.20	2.01	1.00		1.47	1.55		1.33	1.50	
RS/BH	0.96	0.90	0.99		0.95	0.91		0.94	0.57	
ν_{in}^{net}	0.97	0.91	1.00		0.96	0.92		0.96	0.55	
D. Monthly Performance Metric, Quarterly Holding Horizon, Linear Rescaling										
RS	2.59	3.42	2.77	0.65	3.65	2.46	1.20	4.49	0.78	3.71
BH	2.62	3.56	2.81	0.75	3.76	2.55	1.21	4.66	0.90	3.76
RS-BH	-0.02	-0.14	-0.04	-0.10	-0.10	-0.09	-0.01	-0.17	-0.13	-0.04
	[-0.62]	[-2.79]	[-1.36]	[-2.82]	[-2.85]	[-1.88]	[-0.41]	[-2.42]	[-1.20]	[-0.44]
σ_{RS}	5.33	5.75	4.38		4.89	5.61		6.42	8.45	
ρ_{RS}	0.09	0.15	0.00		0.13	0.08		0.03	0.09	
VR	1.01	1.21	1.05		1.17	1.15		1.16	1.13	
RS/BH	0.99	0.96	0.99		0.97	0.96		0.96	0.86	
ν_{in}^{net}	1.00	0.97	0.99		0.98	0.97		0.98	0.85	

Notes: This table reports and decomposes the horizon effects in factor portfolios. Value-weighted CRSP index is from WRDS. Size, value, and momentum factor returns are from Ken French's website. In Panels A and C, the rescaled (RS) performance measures are obtained from daily returns and compared to quarterly buy-and-hold (BH) returns. Panels B and D compare monthly RS returns to quarterly BH returns. RS returns are computed by geometrically (Panels A and B) or linearly (Panels C and D) rescaling the average daily (Panels A and C) or monthly (Panels B and C) portfolio return to a quarterly frequency. BH returns are computed as average of n-day returns, where $n = 1, 21, 63$ corresponds to daily, monthly, and annual periods, respectively. t-statistics for the difference between RS and BH returns are shown in square brackets, and for the case of geometric rescaling are calculated using the Delta method described in the Appendix in the main text . Also reported are the standard deviation σ_{RS} and autocorrelation ρ_{RS} of daily (Panel A) and monthly (Panel B) returns, the relevant variance ratios (VR), as well as the empirically measured bias (RS/BH) and its analytical approximation ν_{in}^{net} from Proposition 4 in the main text. A complete description of the portfolios is provided in Appendix B in the main text.

Table IA5. Horizon Effects in Country Index Portfolios

Country	MSCI Category	RS	BH	RS/BH	RS-BH	Years
Argentina	Frontier	28.71	32.71	0.88	-4.01 [-0.57]	25
Australia	Developed	13.08	12.99	1.01	0.08 [0.11]	43
Austria	Developed	11.64	13.88	0.84	-2.23 [-1.32]	43
Belgium	Developed	13.94	15.28	0.91	-1.34 [-1.39]	43
Brazil	Emergent	35.14	32.58	1.08	2.56 [0.77]	25
Canada	Developed	12.30	12.29	1.00	0.01 [0.02]	43
Chile	Emergent	21.73	23.57	0.92	-1.83 [-1.68]	25
China	Emergent	6.54	7.99	0.82	-1.45 [-0.78]	20
Colombia	Emergent	24.08	26.30	0.92	-2.22 [-1.20]	20
Croatia	Frontier	12.59	16.94	0.74	-4.36 [-1.07]	10
Czech Republic	Emergent	17.44	17.21	1.01	0.23 [0.15]	18
Denmark	Developed	15.36	16.65	0.92	-1.29 [-1.64]	43
Egypt	Emergent	22.14	29.95	0.74	-7.81 [-2.44]	18
Estonia	Frontier	19.60	22.35	0.88	-2.75 [-0.66]	10
Finland	Developed	13.79	16.52	0.83	-2.73 [-1.21]	31
France	Developed	12.98	13.43	0.97	-0.44 [-0.79]	43
Germany	Developed	12.77	13.53	0.94	-0.77 [-0.79]	43
Greece	Developed	9.83	12.74	0.77	-2.91 [-1.35]	25
Hong Kong	Developed	22.34	23.66	0.94	-1.32 [-0.72]	43
Hungary	Emergent	20.37	20.72	0.98	-0.35 [-0.14]	18
India	Emergent	14.61	18.15	0.80	-3.54 [-1.57]	20
Indonesia	Emergent	25.23	30.89	0.82	-5.66 [-1.15]	25
Ireland	Developed	6.05	8.22	0.74	-2.17 [-1.10]	25
Israel	Developed	8.45	9.46	0.89	-1.01 [-1.30]	20
Italy	Developed	8.88	10.19	0.87	-1.30 [-1.12]	43
Japan	Developed	11.65	13.41	0.87	-1.76 [-1.88]	43
Jordan	Frontier	4.80	6.44	0.74	-1.64 [-2.44]	25
Kenya	Frontier	28.60	37.39	0.77	-8.79 [-1.19]	10
Lebanon	Frontier	18.33	18.79	0.98	-0.47 [-0.19]	10
Malaysia	Emergent	13.57	16.57	0.82	-3.00 [-1.28]	25
Mauritius	Frontier	27.62	30.24	0.91	-2.63 [-0.85]	10
Mexico	Emergent	26.98	28.37	0.95	-1.39 [-0.97]	25
Morocco	Emergent	12.19	13.07	0.93	-0.88 [-1.59]	18
Netherlands	Developed	14.29	14.30	1.00	-0.01 [-0.02]	43
New Zealand	Developed	9.21	10.39	0.89	-1.18 [-0.99]	31
Nigeria	Frontier	22.90	25.09	0.91	-2.19 [-0.44]	10
Norway	Developed	16.01	18.96	0.84	-2.95 [-1.73]	43
Pakistan	Frontier	14.38	18.92	0.76	-4.54 [-1.44]	20
Peru	Emergent	24.77	24.72	1.00	0.04 [0.03]	20
Philippines	Emergent	14.04	18.49	0.76	-4.45 [-1.90]	25
Poland	Emergent	26.85	46.62	0.58	-19.76 [-0.94]	20
Portugal	Developed	5.77	6.91	0.83	-1.14 [-1.32]	25
Russia	Emergent	32.85	40.29	0.82	-7.44 [-1.32]	18
Singapore	Developed	16.41	19.16	0.86	-2.76 [-1.44]	43
Slovenia	Frontier	8.78	15.64	0.56	-6.85 [-1.66]	10
South Africa	Emergent	16.94	16.90	1.00	0.04 [0.05]	20
South Korea	Emergent	15.22	18.47	0.82	-3.24 [-1.48]	25
Spain	Developed	12.09	13.05	0.93	-0.96 [-1.22]	43
Sri Lanka	Frontier	13.49	17.19	0.78	-3.70 [-1.12]	20
Sweden	Developed	17.25	17.85	0.97	-0.60 [-0.93]	43
Switzerland	Developed	13.52	13.87	0.97	-0.35 [-0.59]	43
Taiwan	Emergent	13.22	14.64	0.90	-1.42 [-1.06]	25
Thailand	Emergent	17.30	21.50	0.80	-4.20 [-1.16]	25
Turkey	Emergent	30.17	48.30	0.62	-18.13 [-1.66]	25
United Kingdom	Developed	12.98	13.85	0.94	-0.88 [-1.09]	43
United States	Developed	10.97	11.19	0.98	-0.22 [-0.64]	43

Notes: This table reports horizon effects in returns of the MSCI country index portfolios. Geometrically rescaled (RS) returns are average monthly returns compounded 12 months. Buy-and-hold (BH) returns are average annual returns. t-statistic for the difference between RS and BH returns is shown in square brackets. Sample period ends in 2012 and spans the number of years shown in the last column. MSCI category is as of January 2013.

Table IA6. Horizon Effects in Regional Index Portfolios

Regional Index	RS	BH	RS/BH	RS-BH	Years
All Country Americas	11.18	11.66	0.96	-0.48 [-0.93]	25
All Country Asia	3.14	4.61	0.68	-1.47 [-1.91]	25
All Country Asia Excluding Japan	13.03	15.44	0.84	-2.41 [-1.76]	25
All Country Asia Pacific	4.91	5.95	0.82	-1.04 [-1.82]	25
All Country Asia Pacific Excluding Japan	12.25	14.29	0.86	-2.04 [-1.82]	25
All Country EAFE + Emerging Markets	7.74	8.37	0.93	-0.63 [-1.06]	25
All Country Europe	10.62	11.08	0.96	-0.46 [-0.62]	25
All Country Europe + Middle East	6.11	7.42	0.82	-1.31 [-1.04]	25
All Country Far East	4.06	4.98	0.81	-0.92 [-1.61]	25
All Country Far East Excluding Japan	13.05	15.17	0.86	-2.13 [-1.50]	25
All Country Far East Excluding Japan and Hong Kong	11.75	14.32	0.82	-2.57 [-1.58]	25
All Country Pacific	4.83	5.80	0.83	-0.97 [-1.75]	25
All Country Pacific Excluding Japan	12.30	14.15	0.87	-1.85 [-1.65]	25
All Country Pacific Excluding Japan and Hong Kong	11.11	12.98	0.86	-1.87 [-1.67]	25
All Country World	8.72	9.22	0.95	-0.50 [-0.95]	25
All Country World Excluding United States	7.88	8.53	0.92	-0.65 [-1.09]	25
Brazil, Russia, India and China (BRIC)	14.27	18.58	0.77	-4.31 [-2.20]	18
EAFE	11.39	12.10	0.94	-0.71 [-1.53]	43
EAFE + Canada	11.33	11.98	0.95	-0.65 [-1.45]	43
EAFE Excluding Japan	11.87	12.43	0.96	-0.55 [-1.08]	43
EAFE Excluding United Kingdom	11.42	12.22	0.93	-0.80 [-1.62]	43
Emerging Markets	16.08	18.38	0.87	-2.31 [-1.99]	25
Emerging Markets Asia	11.94	15.11	0.79	-3.18 [-2.06]	25
Emerging Markets Eastern Europe	14.05	16.56	0.85	-2.50 [-0.94]	18
Emerging Markets Europe	15.30	18.42	0.83	-3.12 [-1.49]	25
Emerging Markets Europe + Middle East	13.48	16.64	0.81	-3.16 [-1.82]	25
Emerging Markets Europe, Middle East and Africa	13.91	16.05	0.87	-2.14 [-1.24]	16
Emerging Markets Excluding Asia	17.71	20.36	0.87	-2.65 [-1.19]	25
Emerging Markets Far East	11.86	14.76	0.80	-2.90 [-1.63]	25
Emerging Markets Latin America	25.43	27.46	0.93	-2.03 [-1.04]	25
EMU	10.65	11.00	0.97	-0.35 [-0.50]	25
Europe	12.08	12.58	0.96	-0.50 [-0.93]	43
European Union	9.78	10.38	0.94	-0.59 [-0.77]	25
Far East	11.97	13.90	0.86	-1.93 [-1.98]	43
Frontier Markets	10.84	14.49	0.75	-3.65 [-1.41]	10
Frontier Markets Africa	21.23	24.91	0.85	-3.68 [-1.01]	10
Frontier Markets Central and Eastern Europe + CIS	10.77	16.26	0.66	-5.50 [-1.41]	10
Frontier Markets Europe, Middle East and Africa	11.30	14.82	0.76	-3.52 [-1.35]	10
Frontier Markets Excluding Gulf Cooperation Council	12.65	16.51	0.77	-3.86 [-1.25]	10
G7	11.17	11.44	0.98	-0.27 [-0.68]	36
Golden Dragon	8.12	9.61	0.84	-1.49 [-1.13]	16
Nordic	15.70	16.54	0.95	-0.84 [-1.20]	43
North America	11.20	11.40	0.98	-0.20 [-0.58]	43
Pacific	11.71	13.30	0.88	-1.58 [-2.10]	43
Pacific Excluding Japan	13.48	14.62	0.92	-1.14 [-1.39]	43
South East Asia	5.97	9.02	0.66	-3.05 [-1.33]	18
World	10.64	11.03	0.97	-0.39 [-1.07]	43
World Excluding United States	11.47	12.06	0.95	-0.59 [-1.33]	43
Zhong Hua	14.02	16.60	0.84	-2.58 [-1.61]	20

Notes: This table reports horizon effects in returns of the MSCI regional index portfolios. Geometrically rescaled (RS) returns are average monthly returns compounded 12 months. Buy-and-hold (BH) returns are average annual returns. t-statistic for the difference between RS and BH returns is shown in square brackets. Sample period ends in 2012 and spans the number of years shown in the last column.

Table IA7. Horizon Effects in Fama-MacBeth Regressions

	Monthly Horizon						Quarterly Horizon					
	Return-weighted			Size-weighted			Return-weighted			Size-weighted		
	A. Dummy regression using all size categories											
	Coefficient estimates											
	a_L	a_M	a_S	a_L	a_M	a_S	a_L	a_M	a_S	a_L	a_M	a_S
BH	0.90	1.13	2.04	0.88	1.09	1.64	2.91	3.88	6.96	2.77	3.49	5.81
	[4.88]	[4.27]	[5.21]	[5.02]	[5.05]	[4.46]	[4.86]	[4.39]	[4.88]	[4.95]	[5.02]	[4.30]
RS							2.70	3.38	6.11	2.63	3.26	4.91
RS-BH							-0.21	-0.51	-0.85	-0.14	-0.23	-0.90
							[-2.82]	[-2.77]	[-2.26]	[-2.53]	[-2.37]	[-2.54]
	Serial Correlations											
	$a_{L,t}$	$a_{M,t}$	$a_{S,t}$	$a_{L,t}$	$a_{M,t}$	$a_{S,t}$	$a_{L,t}$	$a_{M,t}$	$a_{S,t}$	$a_{L,t}$	$a_{M,t}$	$a_{S,t}$
$a_{L,t-1}$	0.13	0.23	0.28	0.10	0.14	0.25	-0.10	-0.10	-0.05	-0.06	-0.08	-0.04
$a_{M,t-1}$	0.11	0.21	0.29	0.10	0.15	0.28	-0.09	-0.09	-0.04	-0.06	-0.08	-0.02
$a_{S,t-1}$	0.07	0.15	0.22	0.07	0.11	0.22	-0.07	-0.08	-0.06	-0.05	-0.08	-0.05
	B. Dummy regression omitting the large size category and including a constant											
	Coefficient estimates											
	a_L	a_{M-L}	a_{S-L}	a_L	a_{M-L}	a_{S-L}	a_L	a_{M-L}	a_{S-L}	a_L	a_{M-L}	a_{S-L}
BH	0.90	0.23	1.14	0.88	0.21	0.76	2.91	0.98	4.05	2.77	0.72	3.04
	[4.88]	[1.89]	[4.03]	[5.02]	[2.41]	[2.84]	[4.86]	[2.06]	[3.72]	[4.95]	[2.25]	[2.71]
RS							2.70	0.68	3.41	2.63	0.63	2.28
RS-BH							-0.21	-0.30	-0.64	-0.14	-0.10	-0.76
							[-2.82]	[-2.45]	[-1.96]	[-2.53]	[-1.84]	[-2.41]
	Serial Correlations											
	$a_{L,t}$	$a_{M-L,t}$	$a_{S-L,t}$	$a_{L,t}$	$a_{M-L,t}$	$a_{S-L,t}$	$a_{L,t}$	$a_{M-L,t}$	$a_{S-L,t}$	$a_{L,t}$	$a_{M-L,t}$	$a_{S-L,t}$
$a_{L,t-1}$	0.13	0.28	0.29	0.10	0.16	0.26	-0.10	-0.07	-0.01	-0.06	-0.10	-0.03
$a_{M-L,t-1}$	0.01	0.14	0.20	0.06	0.08	0.23	-0.05	-0.04	0.00	-0.03	-0.01	0.05
$a_{S-L,t-1}$	0.00	0.08	0.12	0.03	0.06	0.14	-0.03	-0.08	-0.05	-0.04	-0.06	-0.04
	C. Characteristic regression with demeaned log size as dependent variable											
	Coefficient estimates											
	a_0	$-a_{ME}$		a_0	$-a_{ME}$		a_0	$-a_{ME}$		a_0	$-a_{ME}$	
BH	1.18	0.13		1.13	0.05		4.05	0.48		3.65	0.17	
	[4.52]	[2.98]		[4.71]	[1.79]		[4.55]	[2.91]		[4.71]	[1.83]	
RS							3.53	0.38		3.38	0.14	
RS-BH							-0.52	-0.10		-0.27	-0.03	
							[-2.77]	[-2.14]		[-2.31]	[-1.87]	
	Serial Correlations											
	$a_{0,t}$	$-a_{ME,t}$		$a_{0,t}$	$-a_{ME,t}$		$a_{0,t}$	$-a_{ME,t}$		$a_{0,t}$	$-a_{ME,t}$	
$a_{0,t-1}$	0.21	0.31		0.16	0.16		-0.09	-0.02		-0.09	-0.10	
$-a_{ME,t-1}$	0.04	0.13		0.12	0.12		-0.06	-0.06		-0.03	-0.02	

Notes: This table reports the results of Fama-MacBeth regressions of returns of stocks i in period t. The regressions in the three panels are

$$R_{int} - 1 = a_{Lnt} d_{10,it} + a_{Mnt} d_{2:9,it} + a_{Snt} d_{1,it} + \epsilon_{int}, \tag{33}$$

$$R_{int} - 1 = a_{Lnt} + a_{M-L,nt} d_{2:9,it} + a_{S-L,nt} d_{1,it} + \epsilon_{int}, \text{ and} \tag{34}$$

$$R_{int} - 1 = a_{0nt} + a_{ME,nt} ME_{it} + \eta_{int}, \tag{35}$$

respectively, where R_{int} is the return of stock i in month (in the left set of columns) or quarter (in the right set of columns) $t \in \tau$, $d_{1,it}$, $d_{10,it}$, and $d_{2:9,it}$ are the indicators for membership in the smallest size decile, largest deciles, and in deciles two through nine, respectively, and ME_{it} is the logarithm of market capitalization. All explanatory variables are measured at the beginning of each calendar year. Results are obtained using weighting matrices based on prior month return (Return-weighted) and market equity (Size-weighted). t-statistics in square brackets are computed following Newey and West (1987). The row labeled RS shows the coefficients from monthly horizon regressions multiplied by three. Also shown are serial correlations of regression coefficients, where the coefficients in the rows are lagged: For example, $Corr(a_{St}, a_{Lt-1}) = 0.28$ in the monthly horizon Panel A regression when using the return-weighted approach.

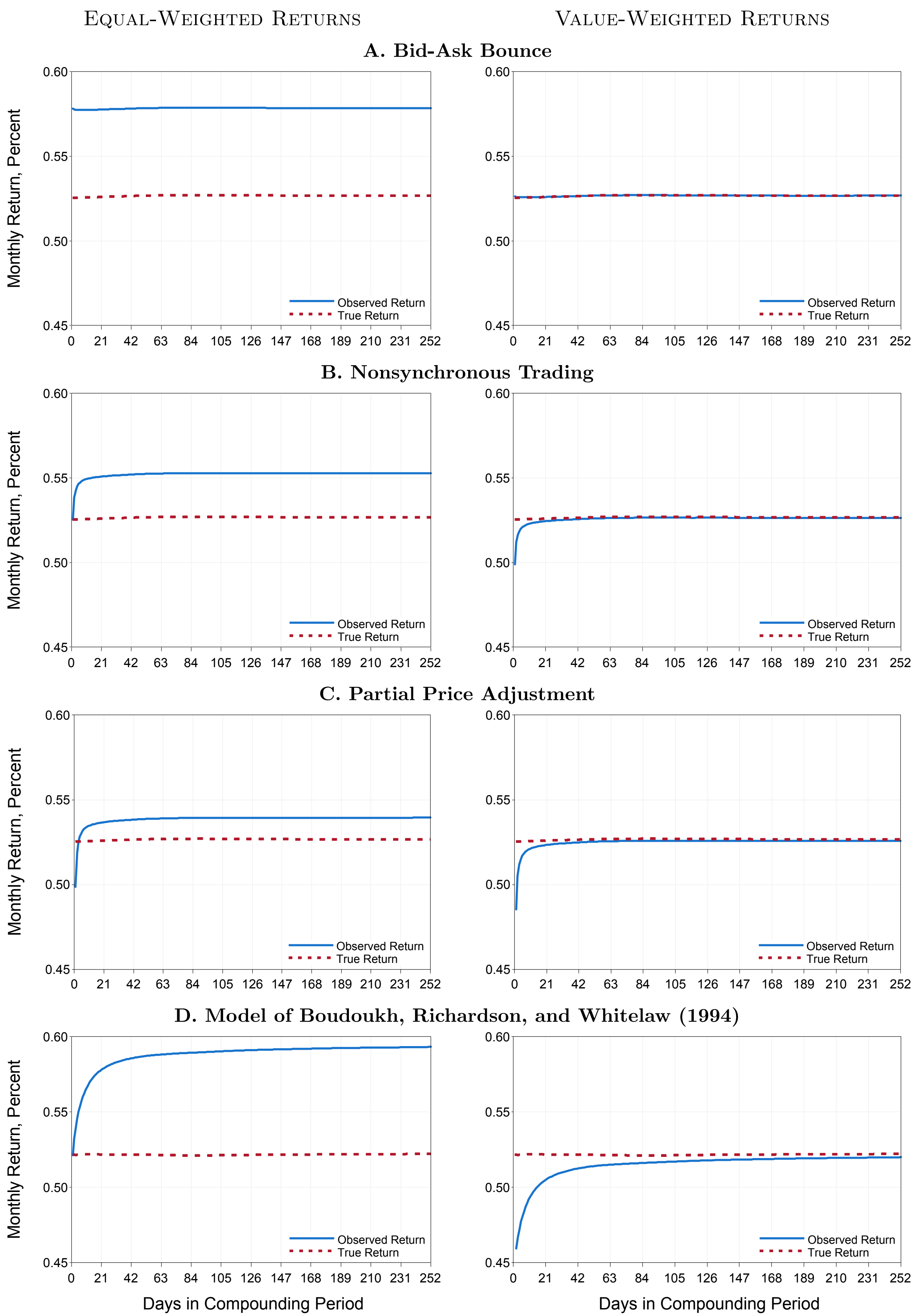

Figure IA1. This figure plots average rolling n-day buy-and-hold returns scaled to a monthly equivalent obtained from models of bid-ask bounce, nonsynchronous trading, partial price adjustment of Section 1, and from the model of Boudoukh, Richardson, and Whitelaw (1994). The number of days n in a compounding horizon is shown on the x-axis.

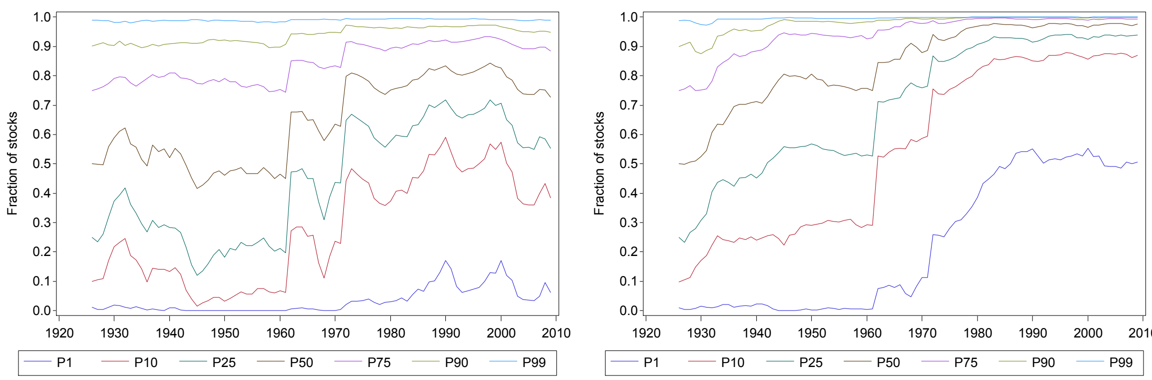

Figure IA2. This figure plots cumulative proportion of stocks that fall into 1926 market capitalization breakpoints, adjusted for value-weighted (left) or equal-weighted (right) market returns.

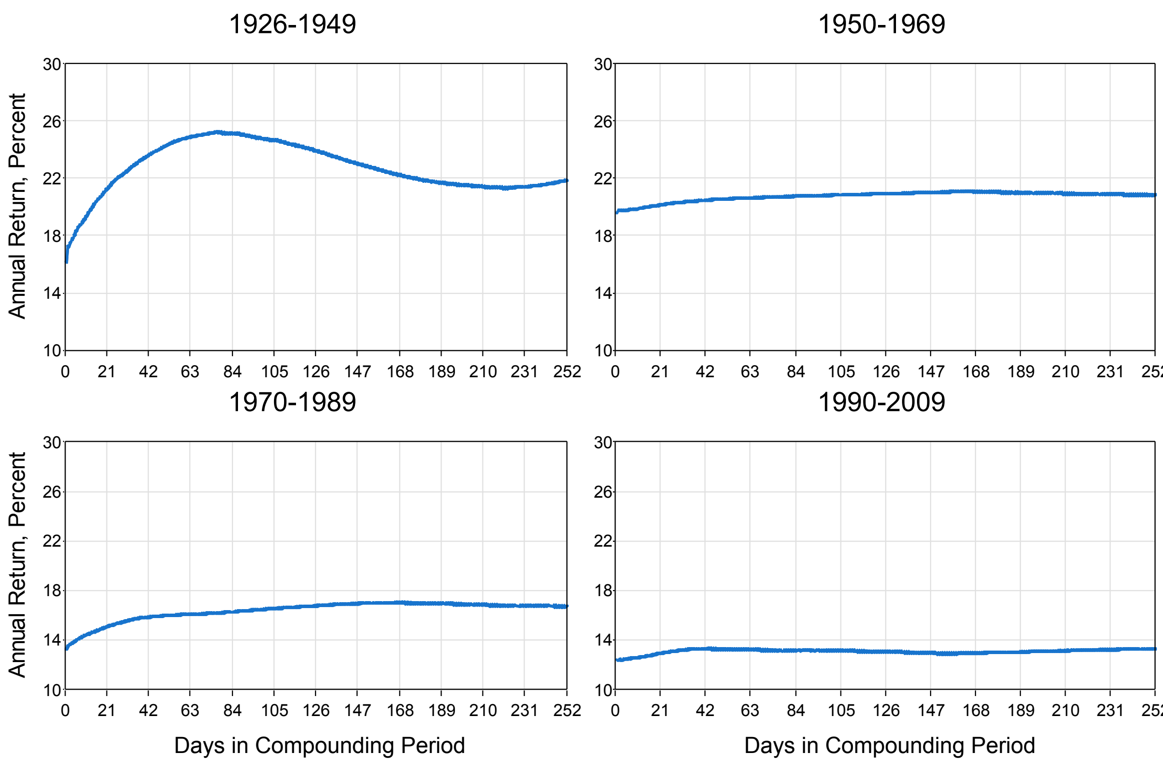

Figure IA3. This figure plots for the small-stock portfolio average rolling n-day buy-and-hold returns scaled to an annual equivalent, $\bar{R}_{in,252} - 1 = [\mathbb{E}\,(R_t \cdots R_{t+n-1})]^{252/n} - 1$ against the buy-and-hold horizon n in four subperiods. The small-stock portfolio contains all stocks that fall between the 1st and 10th percentile breakpoints, which are obtained in 1926 and adjusted for market returns.